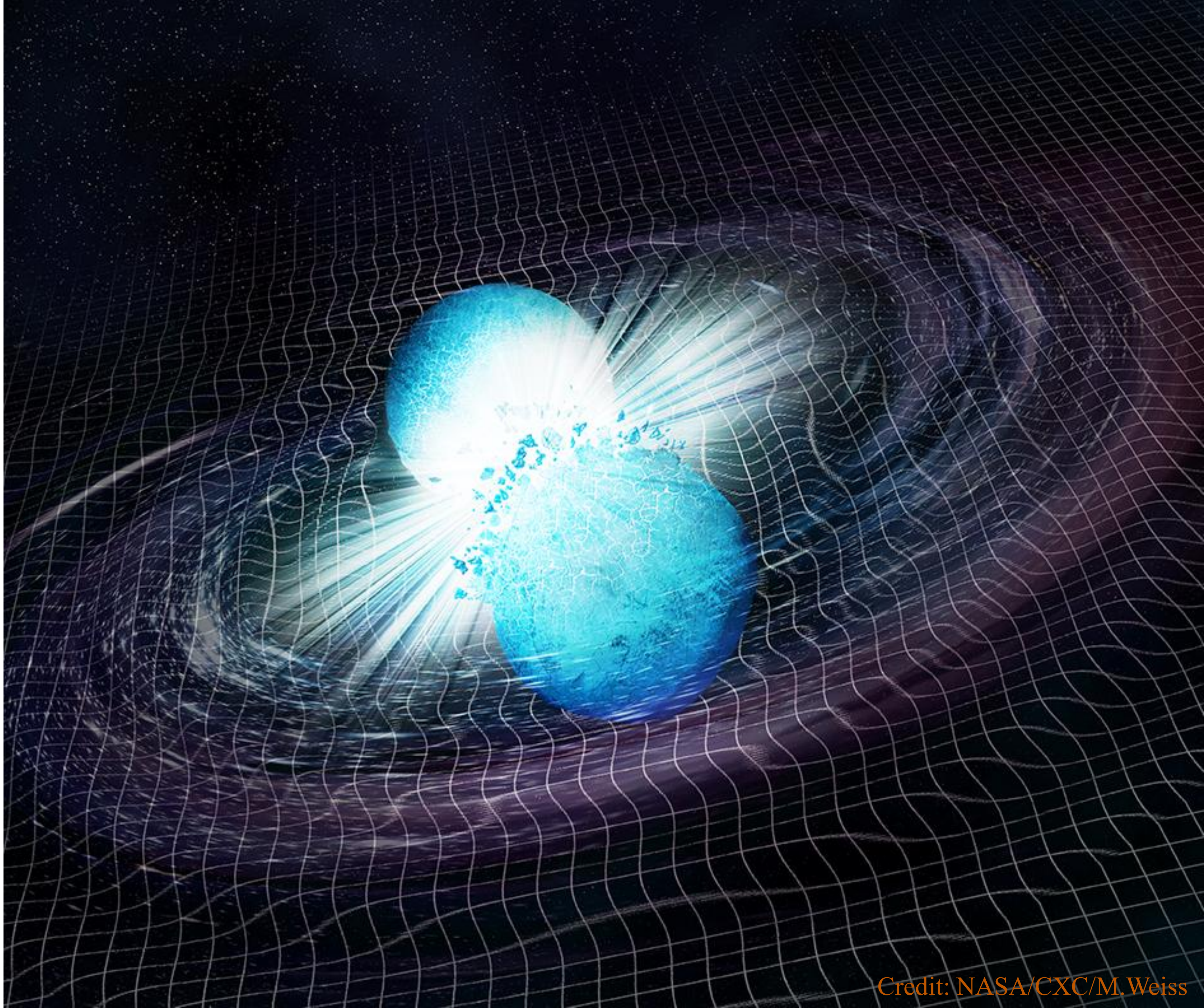
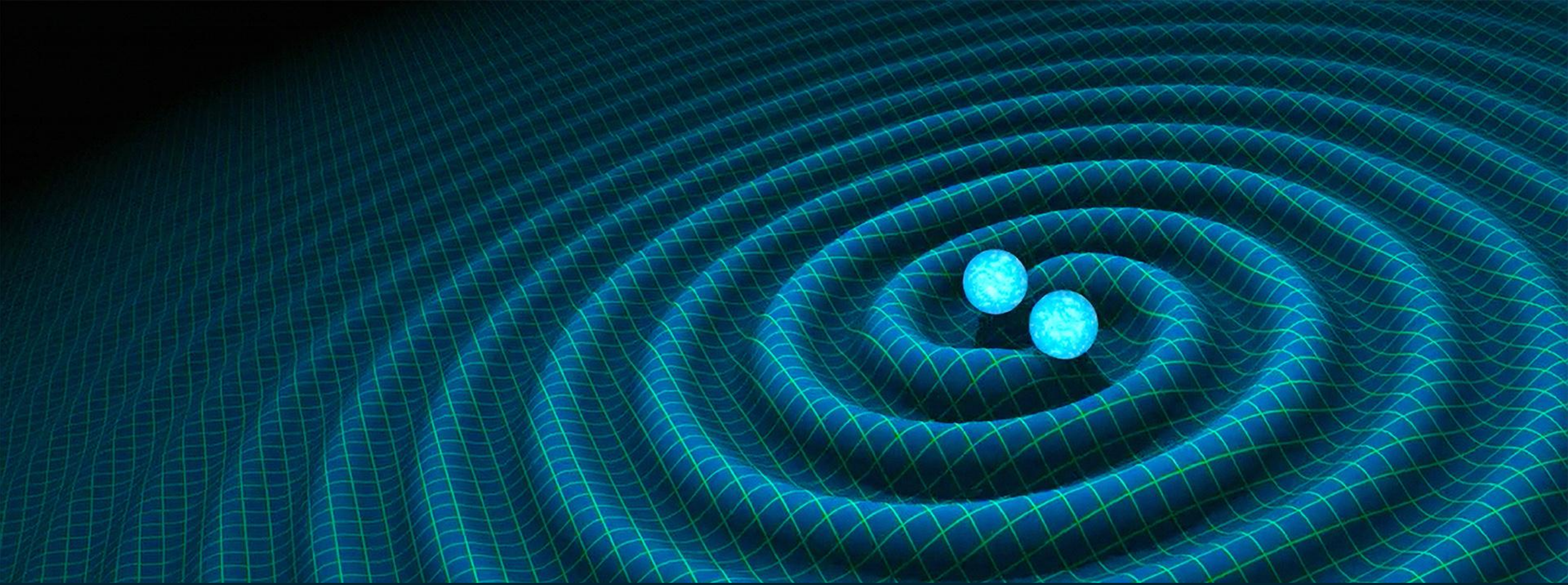
Introduction to Standard Sirens Cosmology

Alberto Salvarese



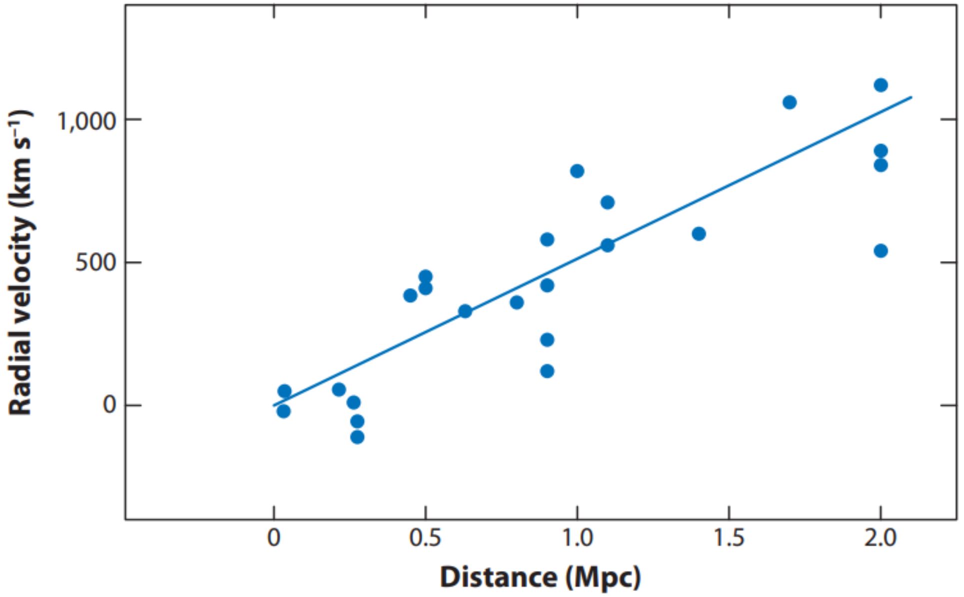


Introduction to Standard Sirens Cosmology

The expanding Universe

In 1929 Edwin Hubble provided evidence that our Universe is expanding

(E. Hubble, 1929; G. Lemaître, 1927)



(Freedman & Madore, 2010)

Hubble's law (z $\leq$ 0.1): $v = cz \propto D$

Hubble constant: $H_0 = \frac{v}{D}$

The expanding Universe

At higher redshifts the relation also depends on the energy content of the Universe

$$H_0 = \frac{c(1+z)}{D_L} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_k(1+z')^2 + \Omega_\Lambda}}$$

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Why is measuring H_0 important?

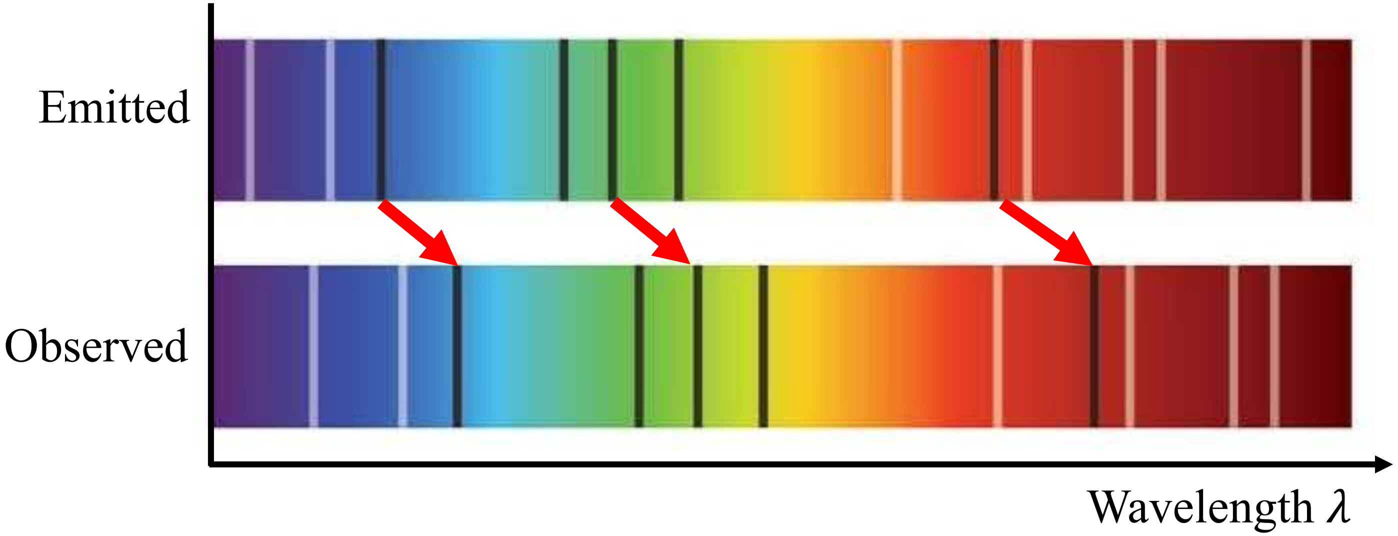
H_0 tells us how fast the Universe is expanding: age of the Universe and its expansion history

Direct probe of ΛCDM and other cosmological models

The expanding Universe: experiments

Direct measurements: directly measure D_L and z (cepheids, SN Ia, etc)

- z from spectroscopy



$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

(Freedman & Madore, 2010)

The expanding Universe: experiments

Direct measurements: directly measure D_L and z (cepheids, SN Ia, etc)

- z from spectroscopy
- $m - M = 5\log(D_L) - 5$

(Freedman & Madore, 2010)

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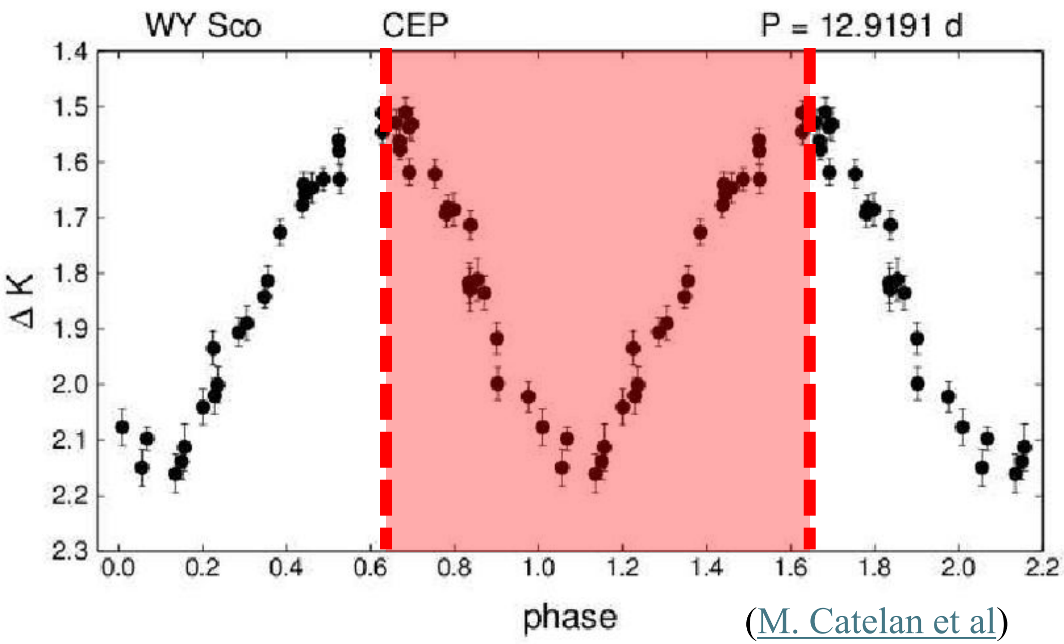


(M. Catelan et al)

Period-luminosity relation

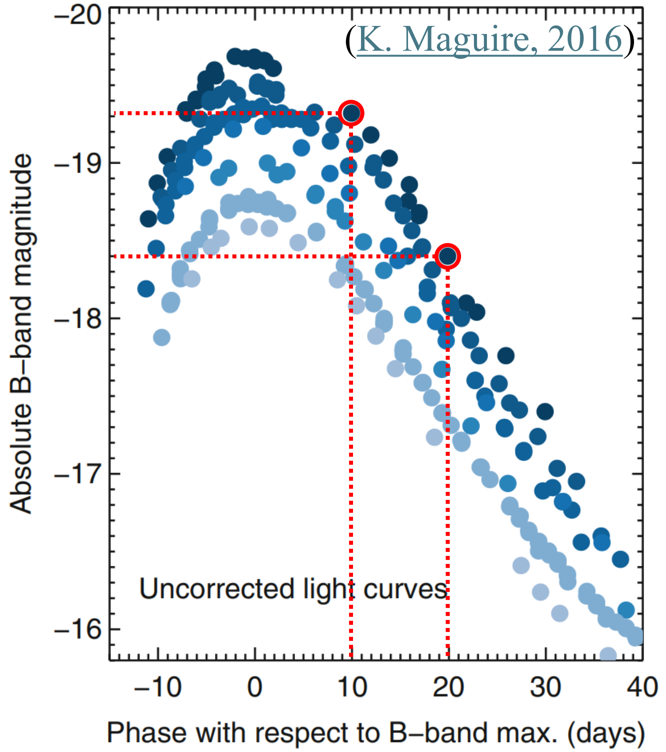
(Freedman & Madore, 2010)

The expanding Universe: experiments

Direct measurements: directly measure D_L and z (cepheids, SN Ia, etc)

- z from spectroscopy
- $m - M = 5\log(D_L) - 5$

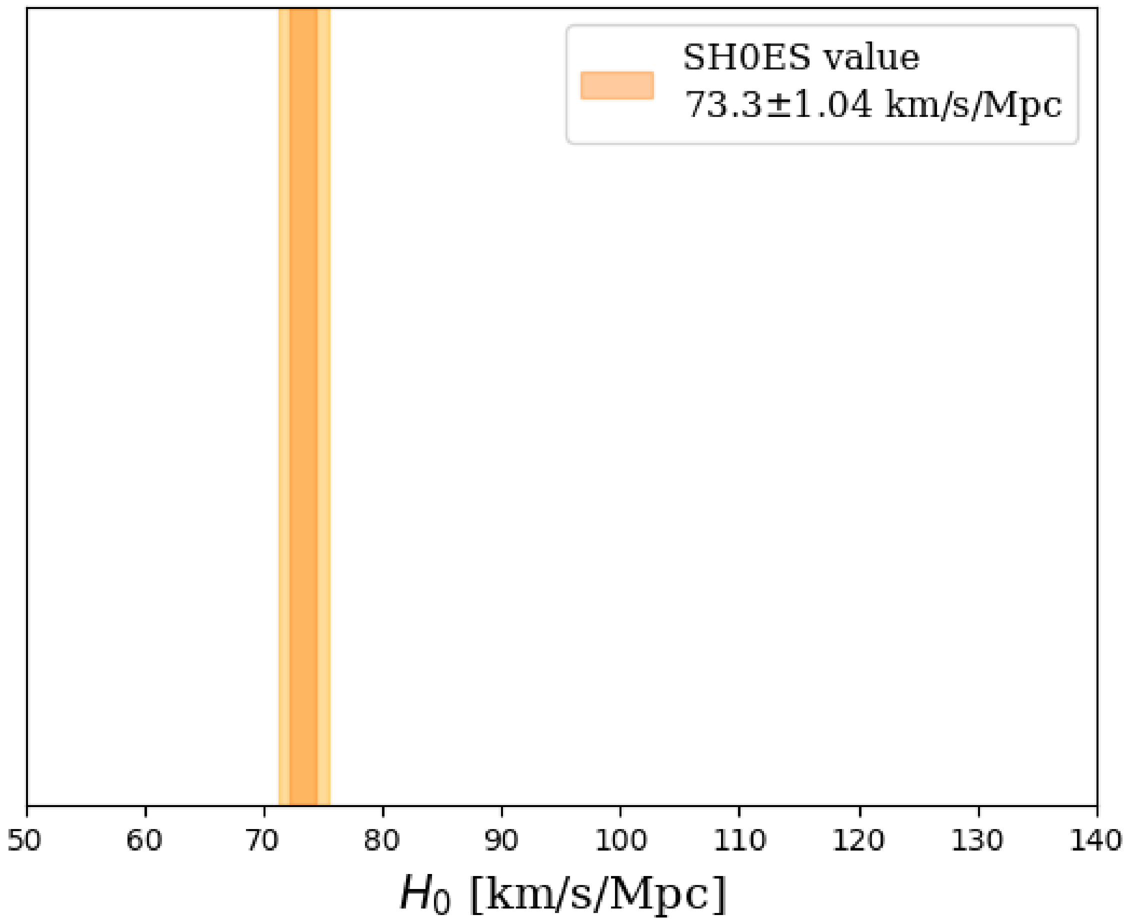
Standard candles



Magnitude at the peak – decline rate relation

(Freedman & Madore, 2010)

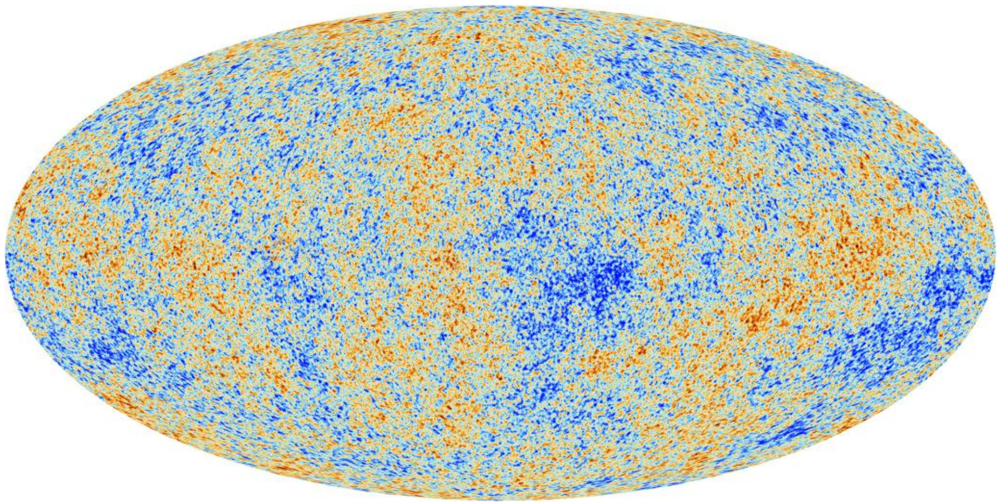
The expanding Universe: experiments



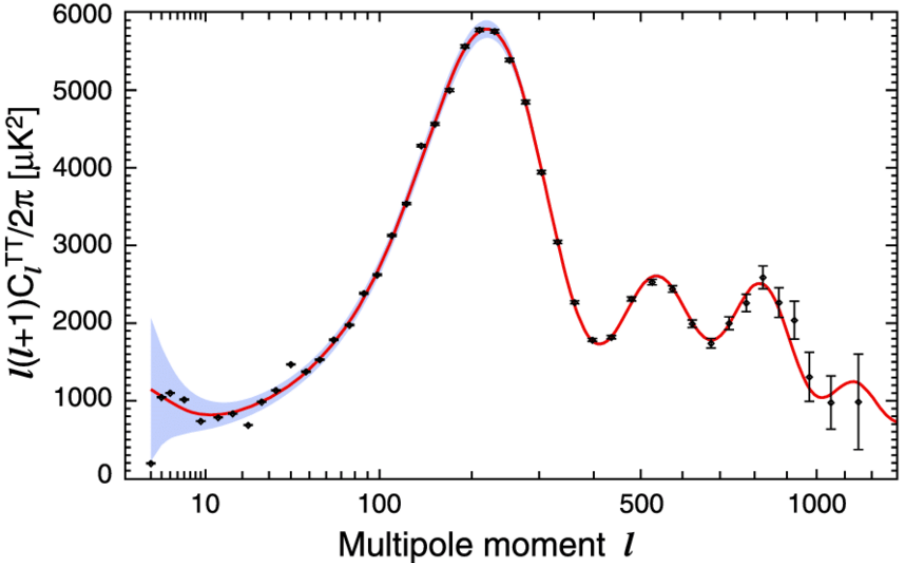
(A. G. Riess et al., 2022)

The expanding Universe: experiments

Indirect measurements: CMB, BAO, etc



(Plank collaboration, 2013)

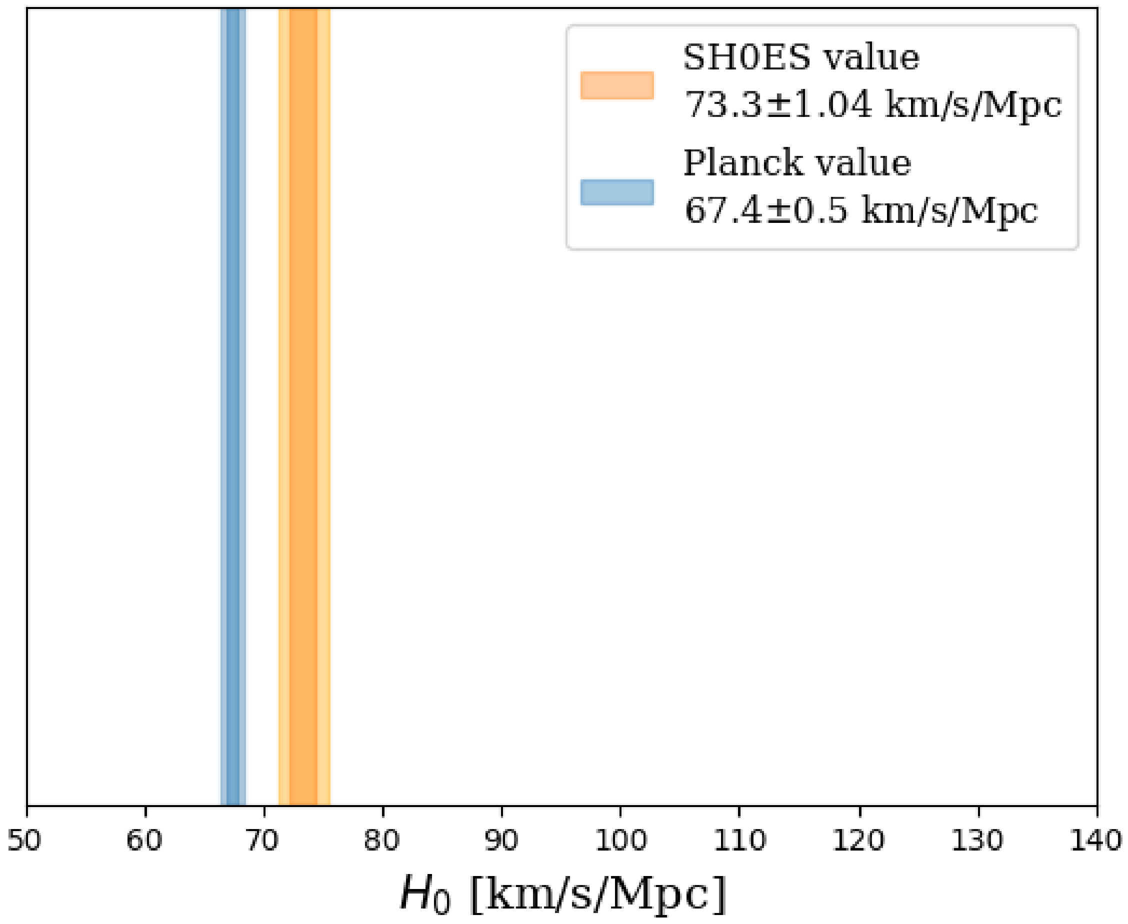


(WMAP Collaboration, 2011)

$H_0 = f(\vec{\Theta})$ assuming ΛCDM model

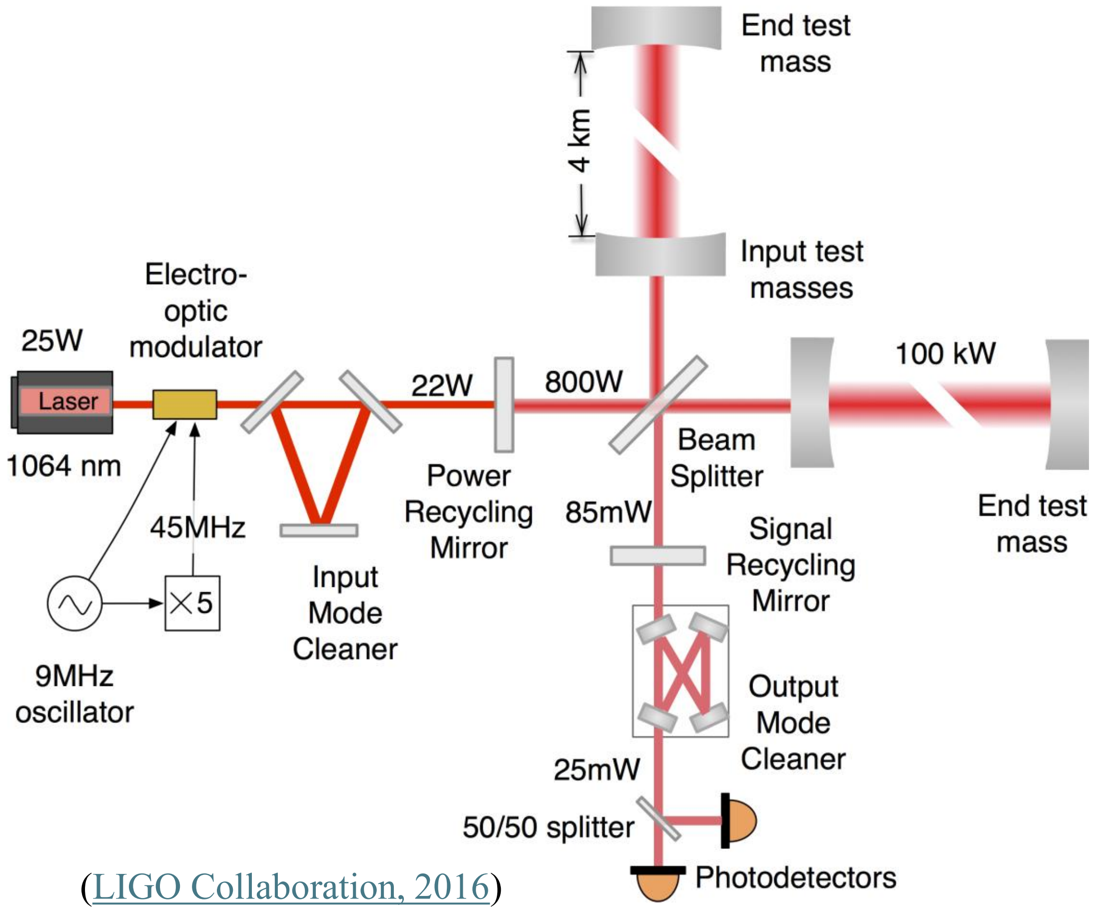
(Kamionkowski & Riess, 2023)

The expanding Universe: 5σ tension

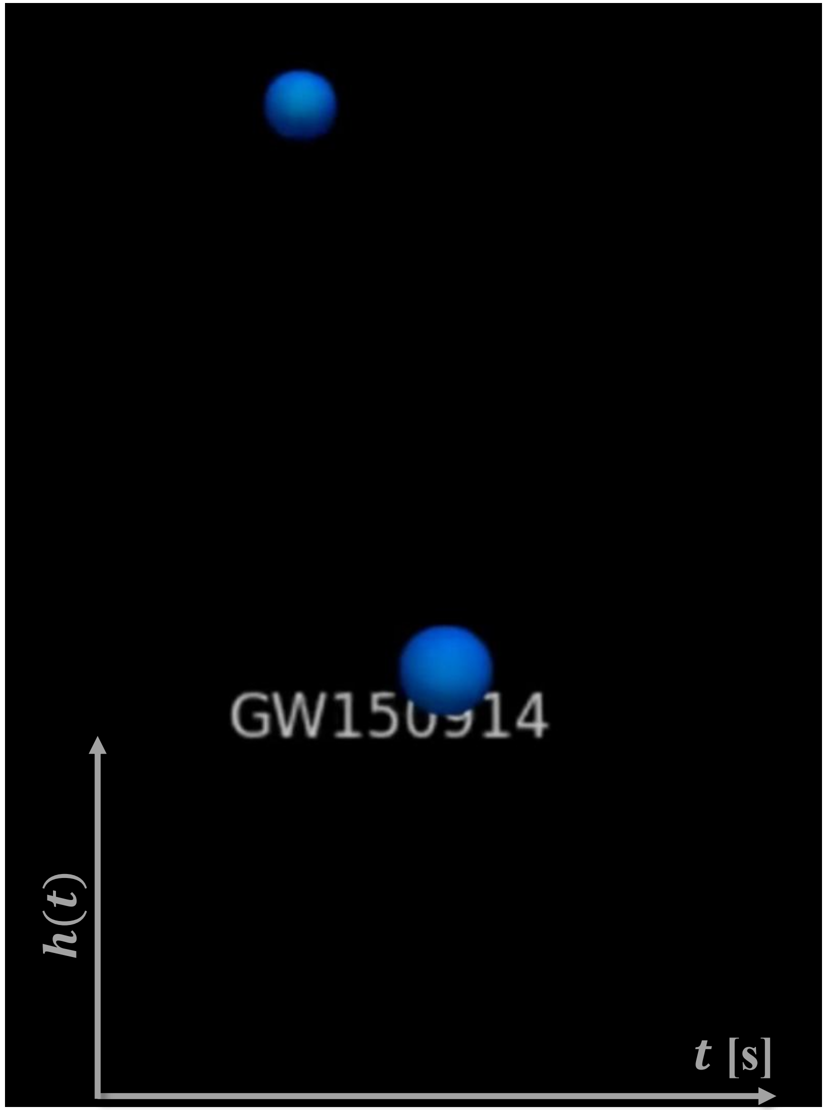


(Planck collaboration, 2020)

Gravitational waves



(LIGO Collaboration, 2016)



(LIGO Caltech)

Amplitude of the signal: $h = \frac{\Delta L}{L}$

Hubble constant with GW

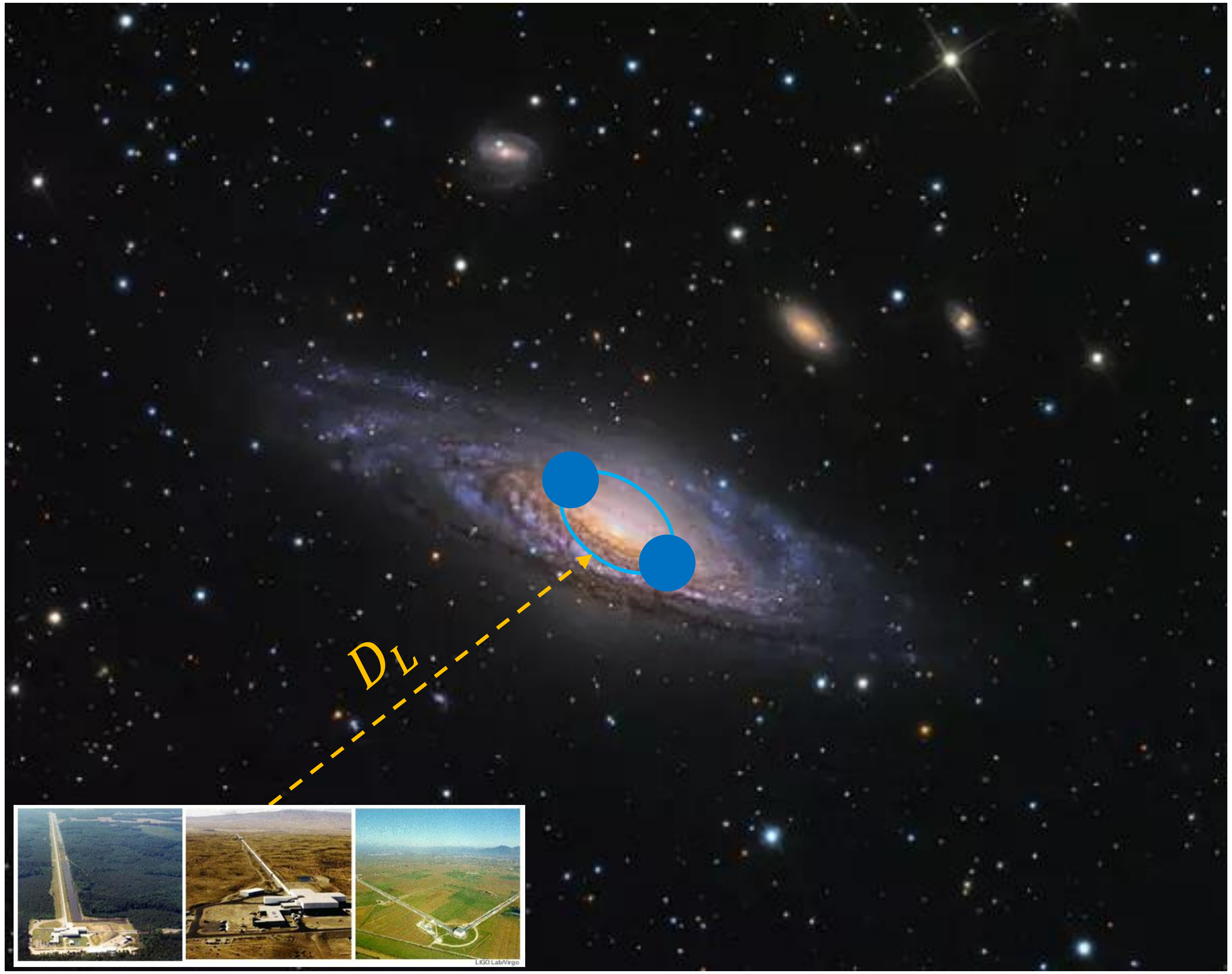
$$h(f) = F_+ h_+(f) + F_\times h_\times(f)$$

$$h_+(f) \propto \frac{M_z^{5/6}}{D_L} (1 + \cos^2(\iota)) f^{-\frac{7}{6}} e^{i\phi(M_z, f)}$$

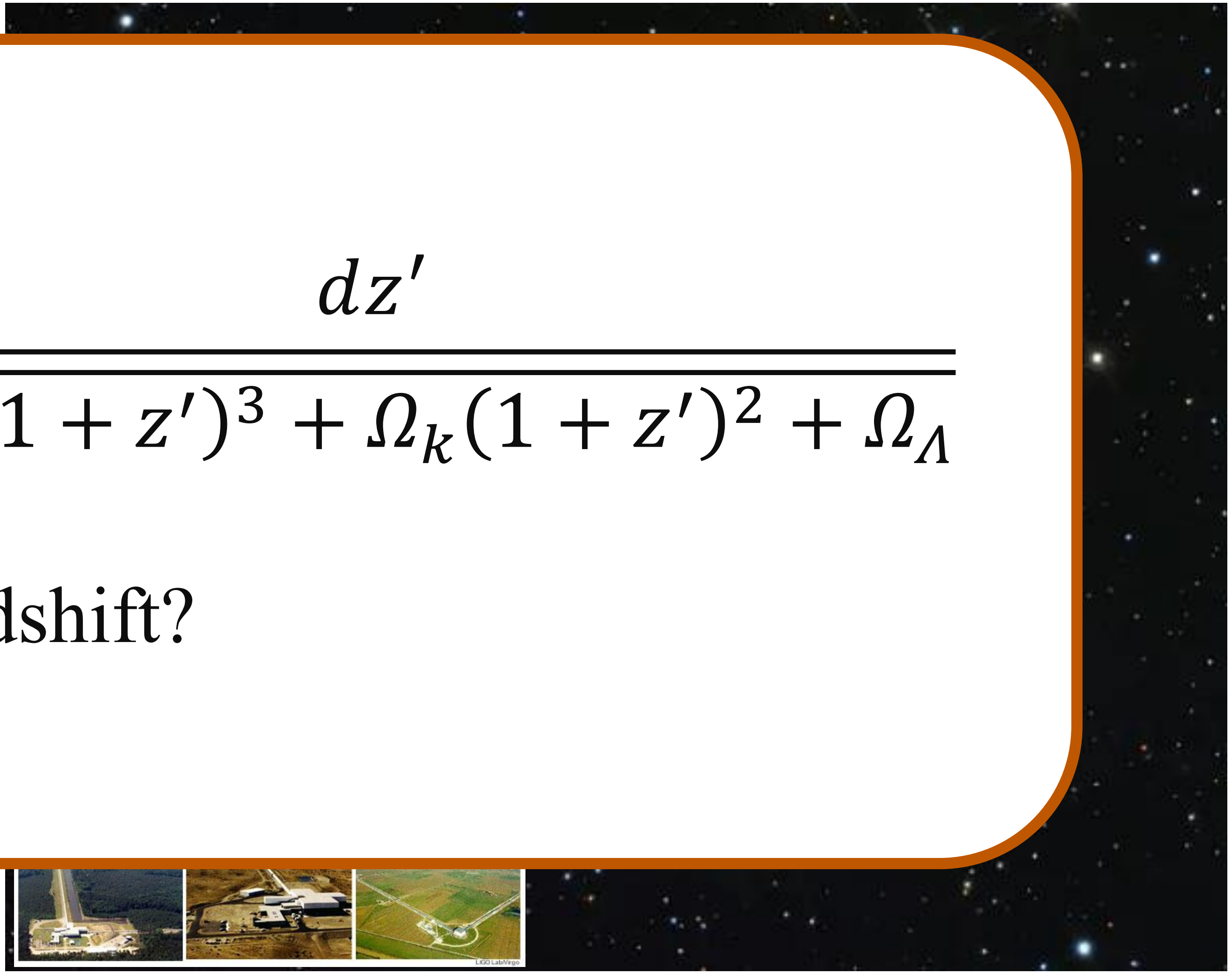
$$h_\times(f) \propto \frac{M_z^{5/6}}{D_L} \cos(\iota) f^{-\frac{7}{6}} e^{i\phi(M_z, f) + \frac{i\pi}{2}}$$

Luminosity distance D_L from compact binaries gravitational wave signal

Standard sirens



Hubble constant with GW

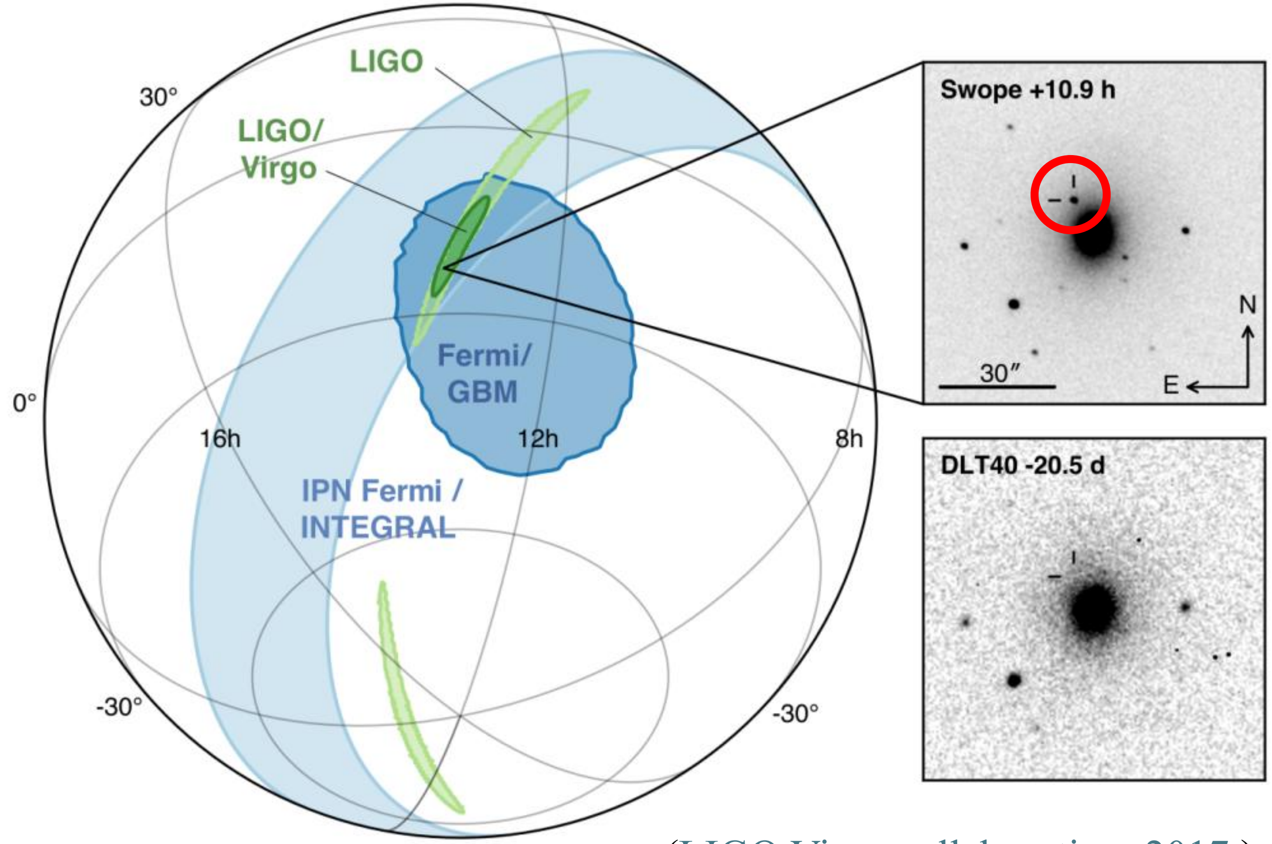


$$H_0 = \frac{c(1+z)}{D_L} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_k (1+z')^2 + \Omega_\Lambda}}$$

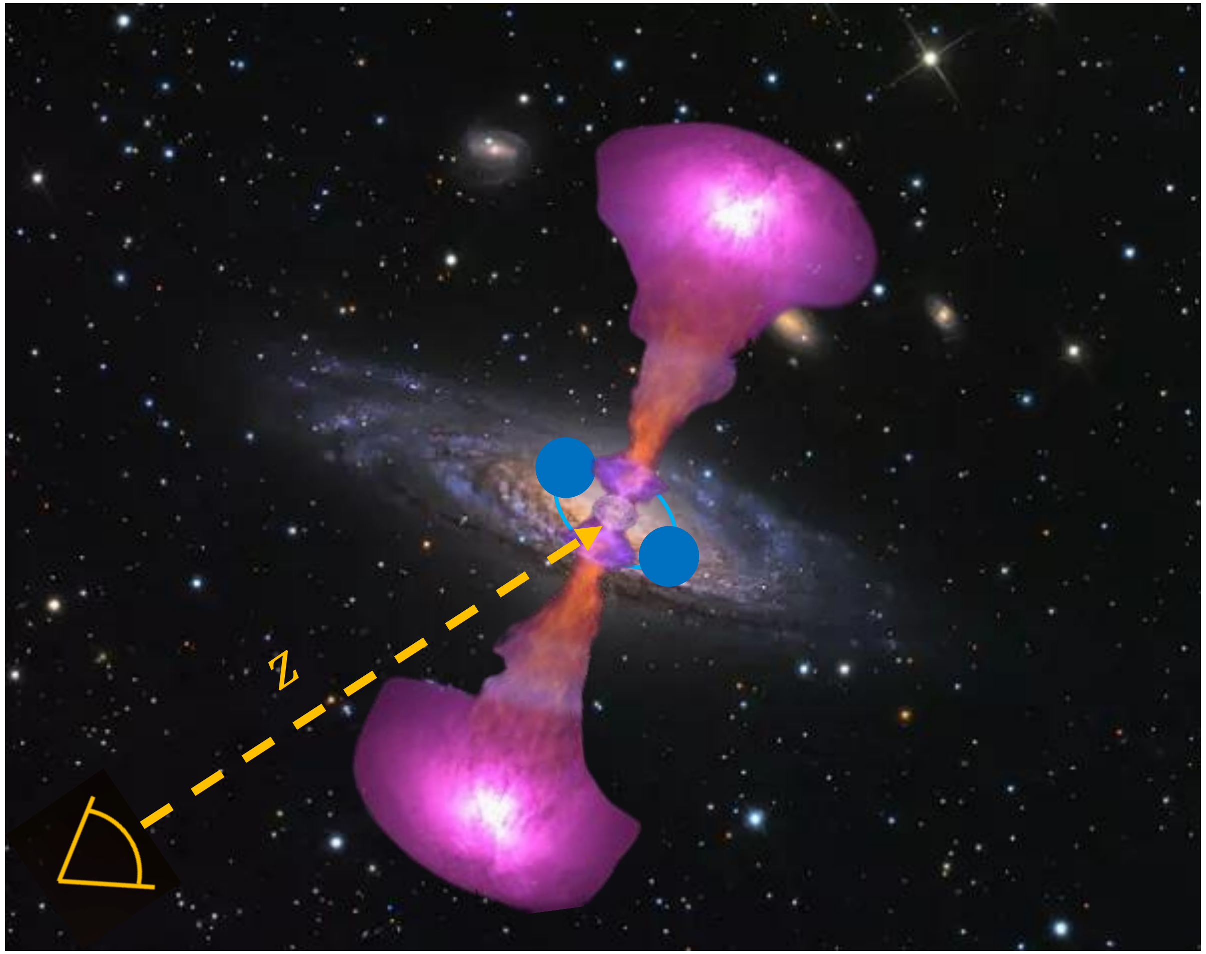
Redshift?

Hubble constant with GW: bright sirens

Bright sirens: compact binary merger with EM emission (BNS, NSBH)

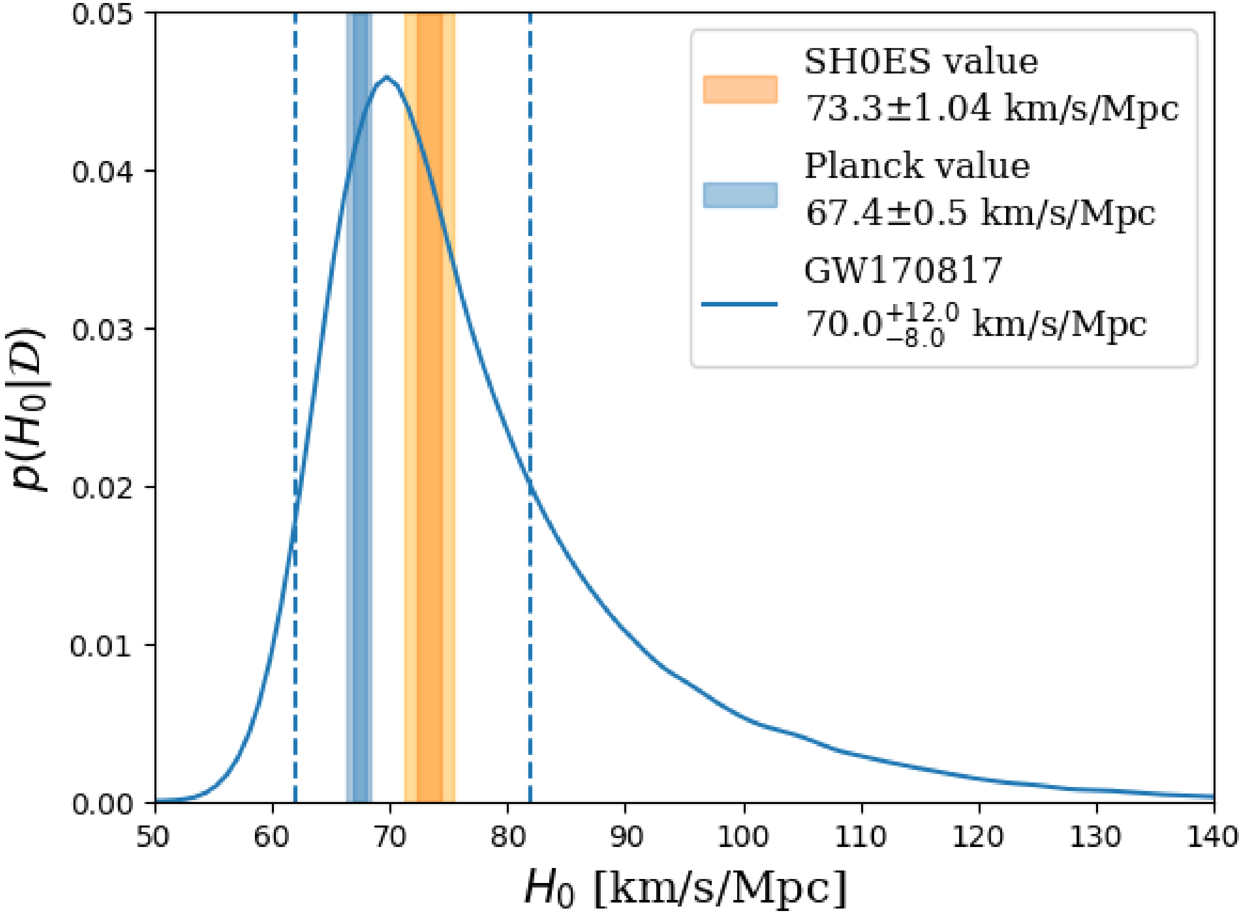


(LIGO Virgo collaboration, 2017)



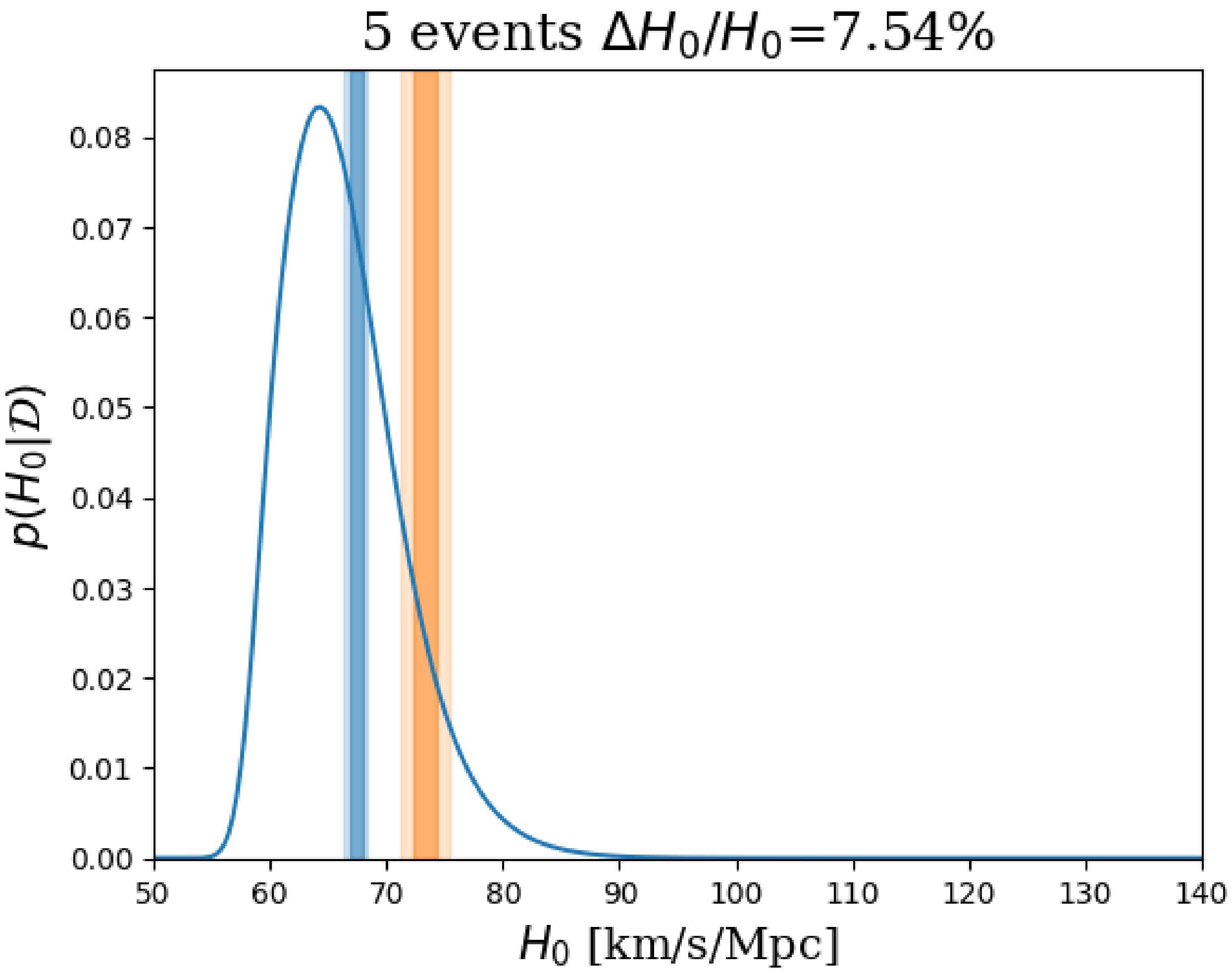
(B. F. Schutz, 1986)

Bright sirens estimate



(LIGO Virgo collaboration, 2017)

Bright sirens estimate



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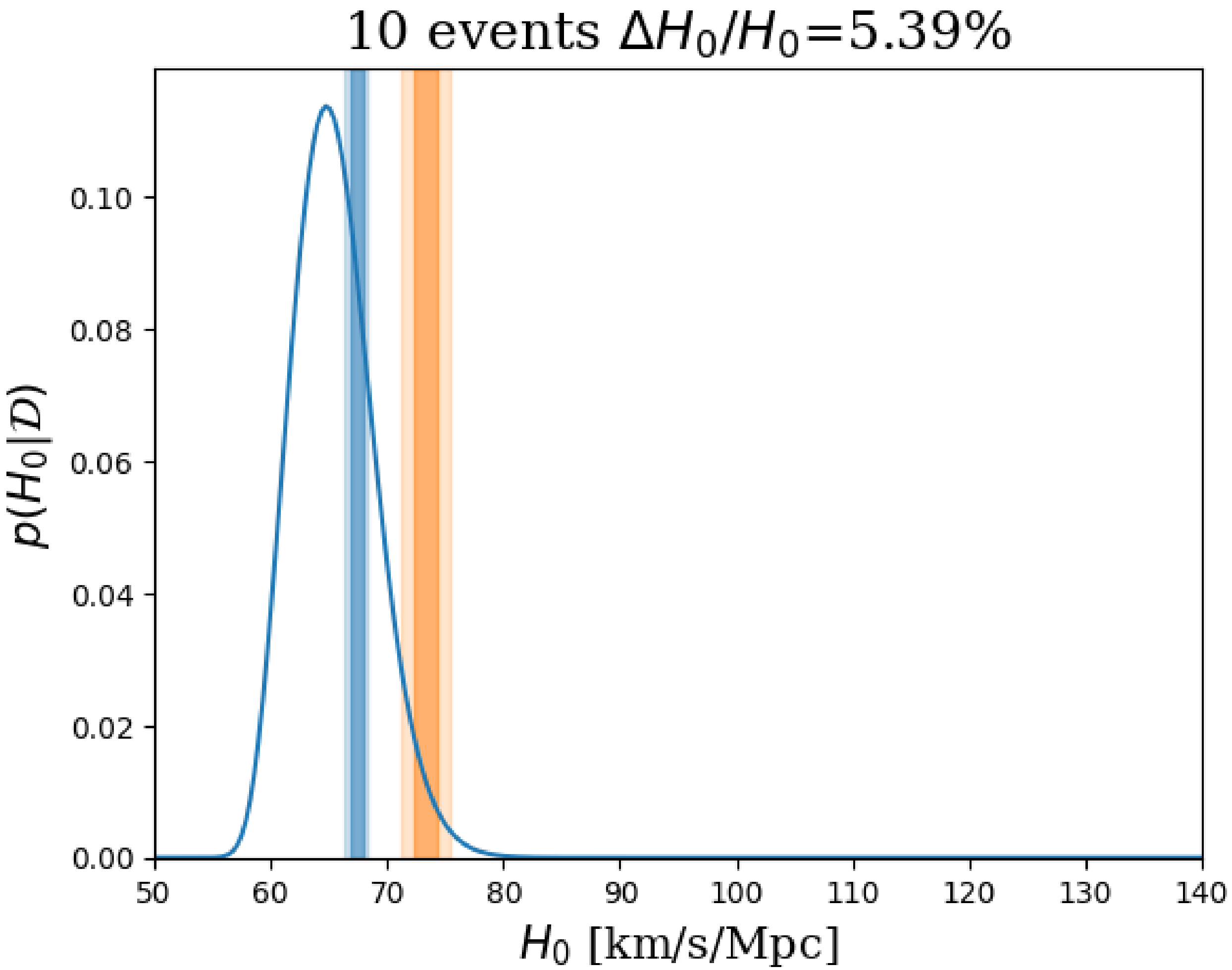
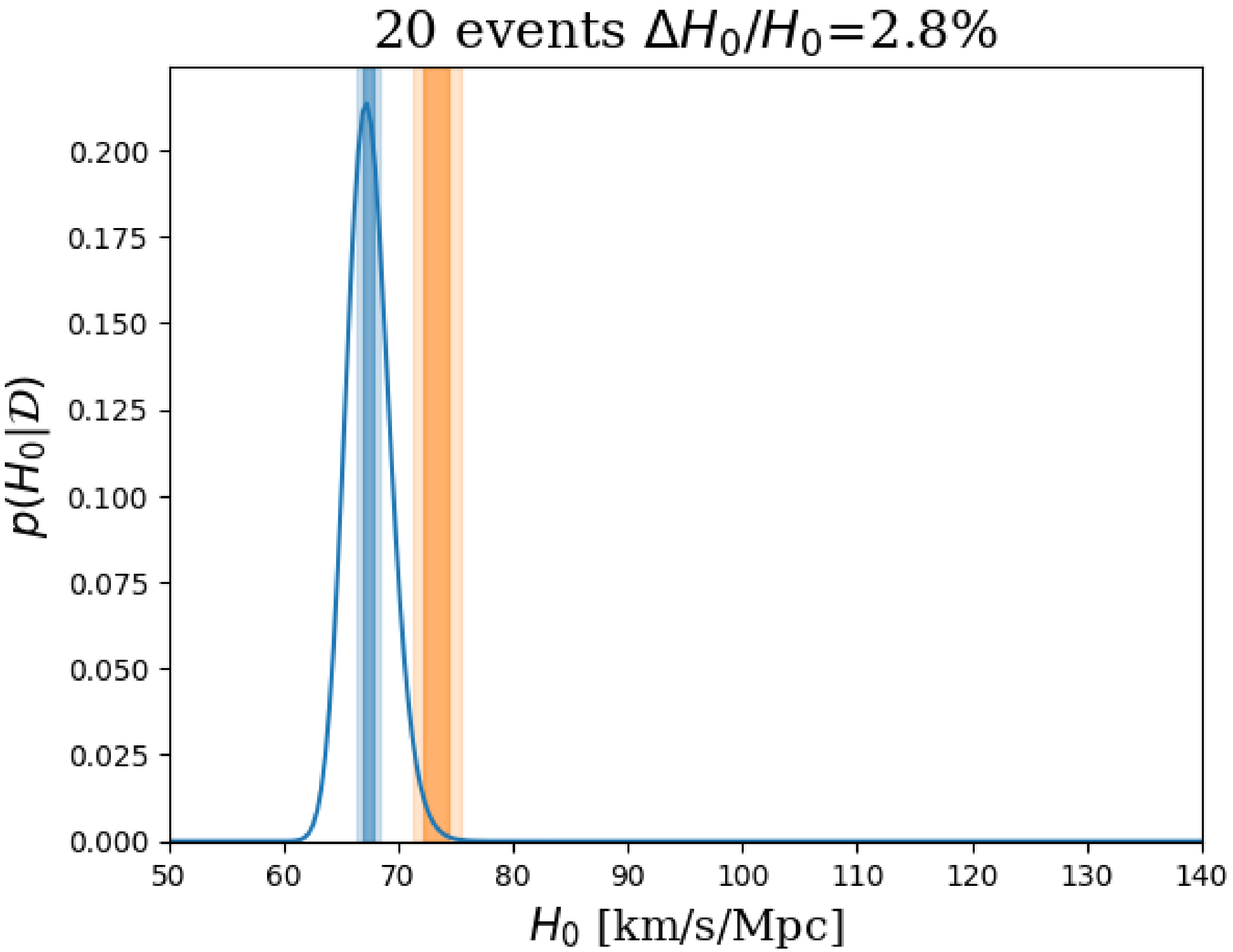


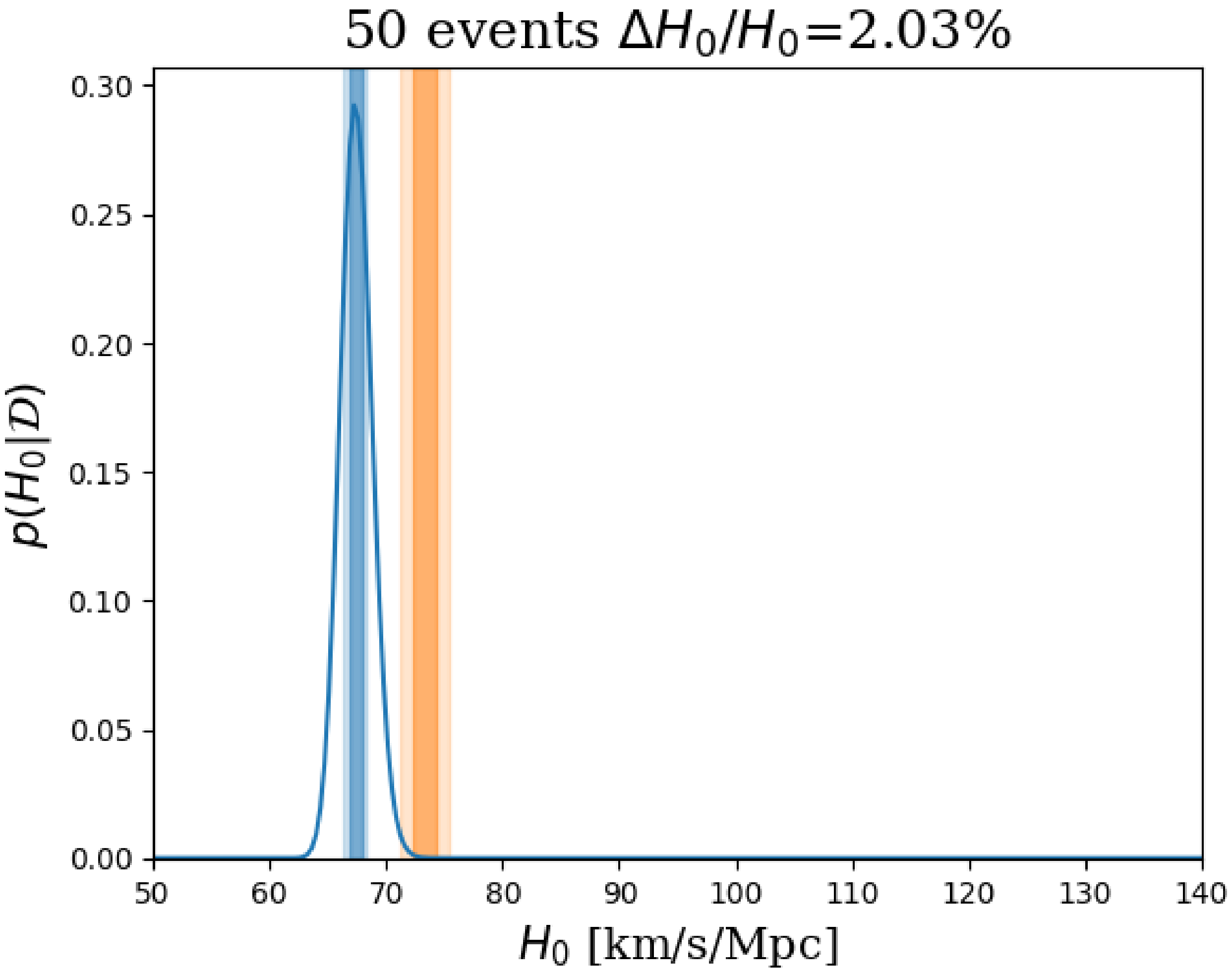
Bright sirens estimate



Bright sirens estimate

Strengths

Only ~ $O(50)$ events needed to solve the tension (Chen et al., 2018)

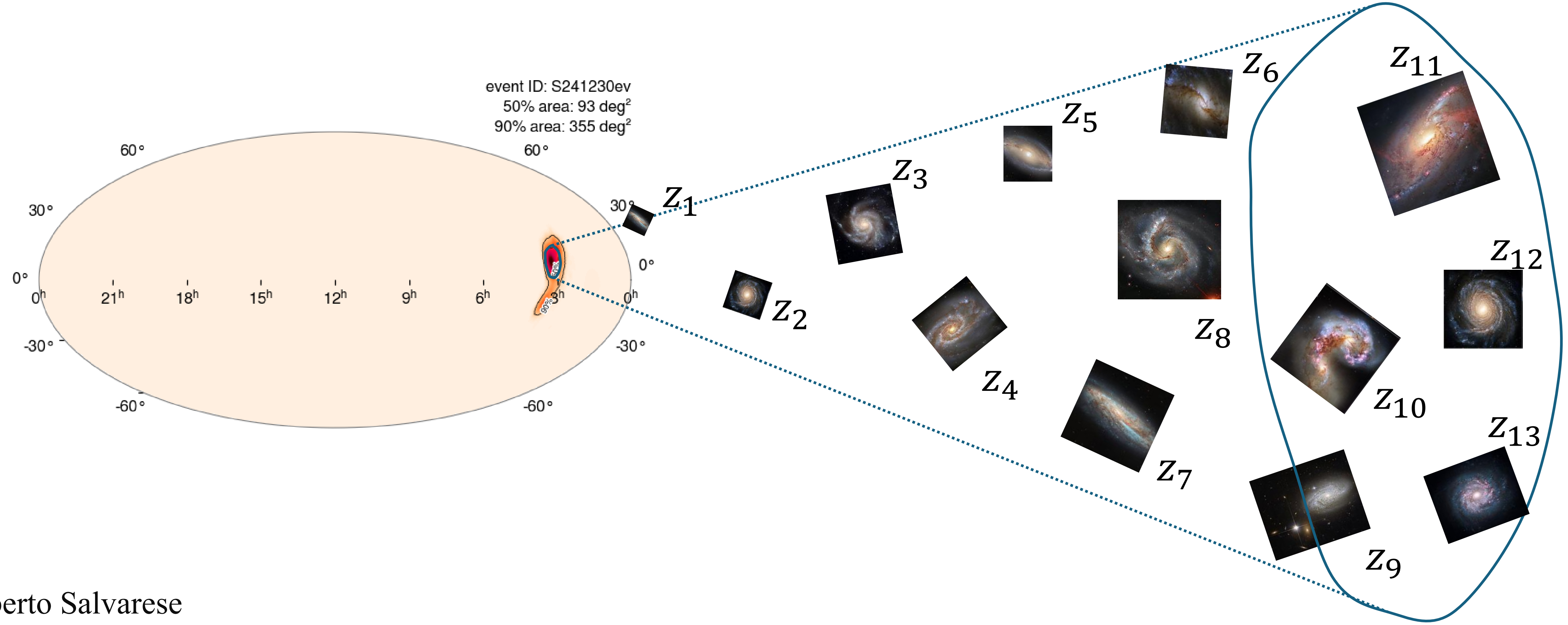


Weaknesses

- Not very massive: difficult to detect, and only in the local Universe
- Only one bright siren so far

Hubble constant with GW: dark sirens

Dark sirens: compact binary merger with only GW emission. Once the event is localized in the sky, z is inferred through galaxy catalogue (W. Del Pozzo, 2012)



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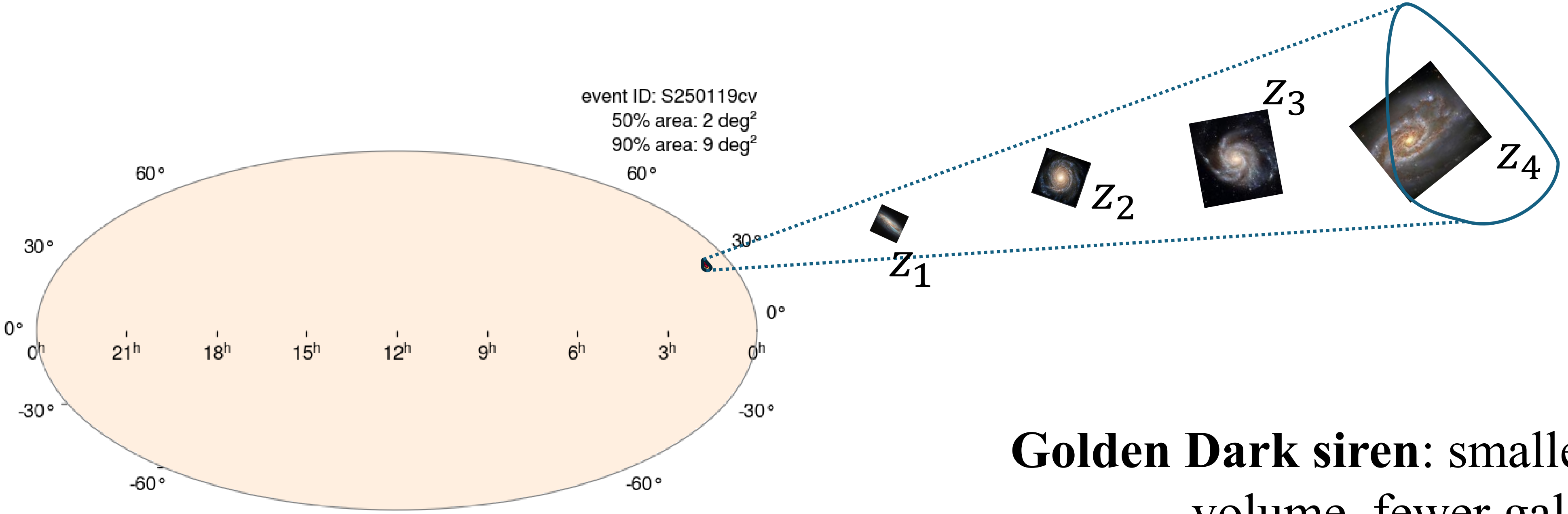
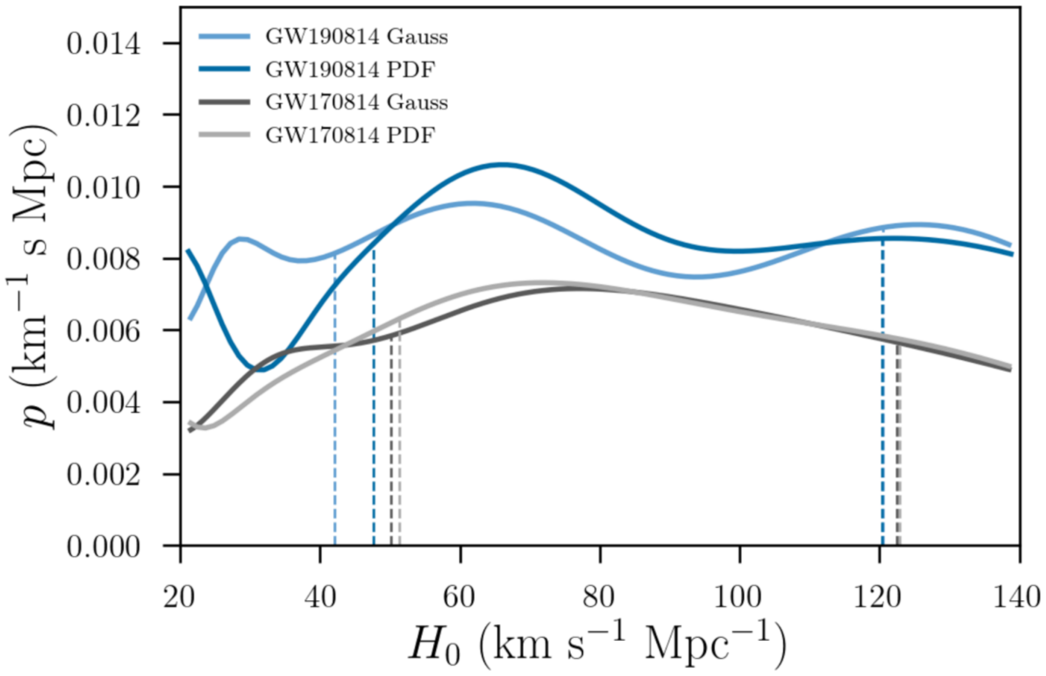
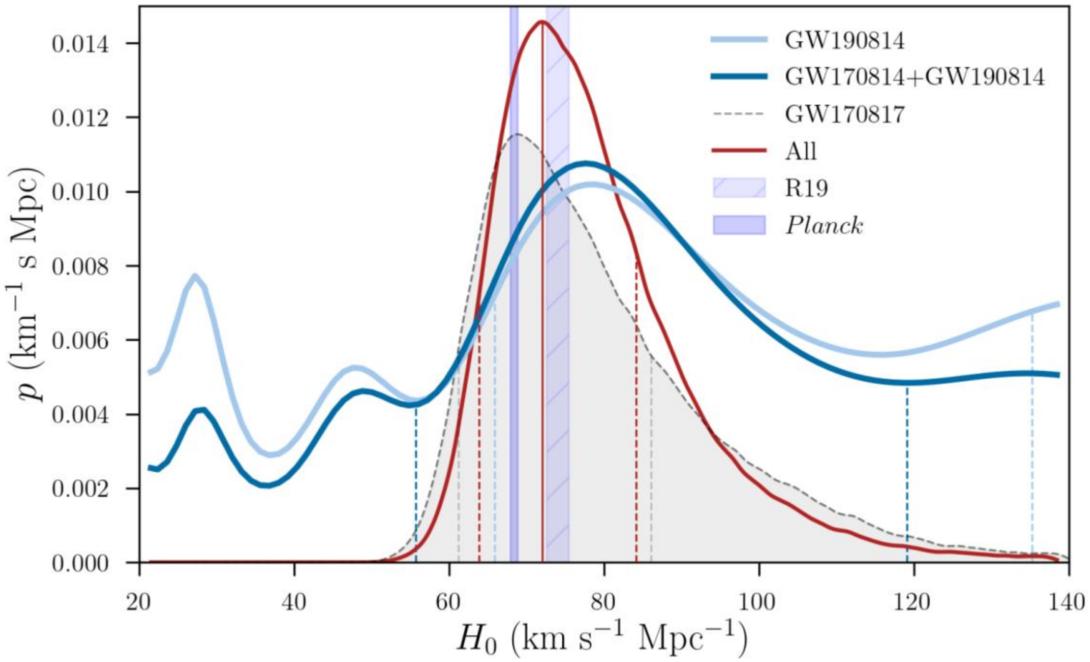


Golden Dark siren: smaller localization volume, fewer galaxies

Hubble constant with GW: dark sirens

Combine bright sirens measurements (GW170817) with golden dark siren estimates

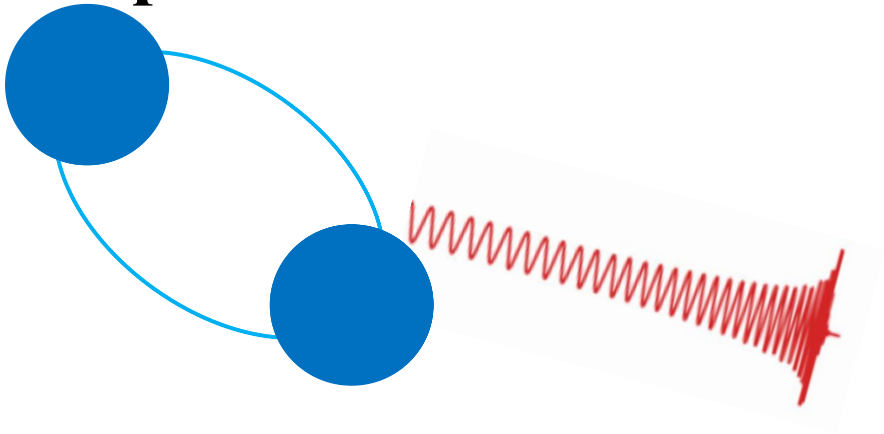




(A. Palmese et al., 2020)

Hubble constant with GW: spectral sirens

Spectral sirens: redshift inferred by the estimated mass (Chernoff & Finn, 1993)



Hubble constant with GW: spectral sirens

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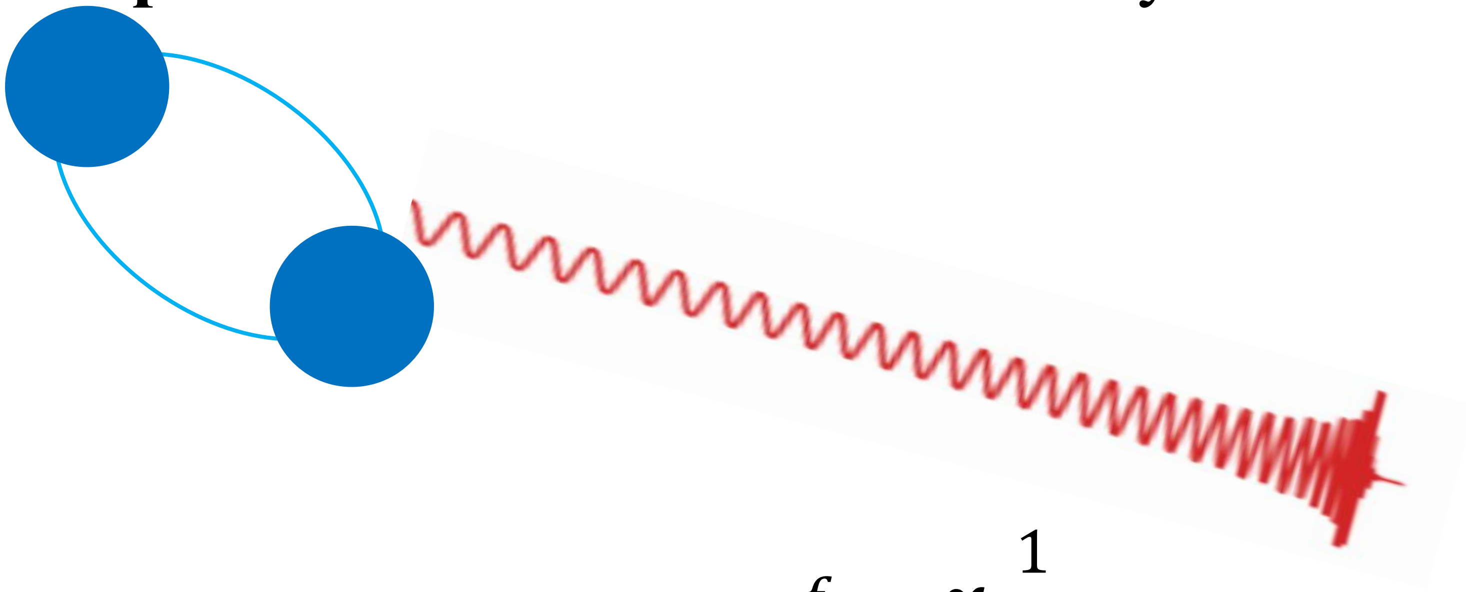
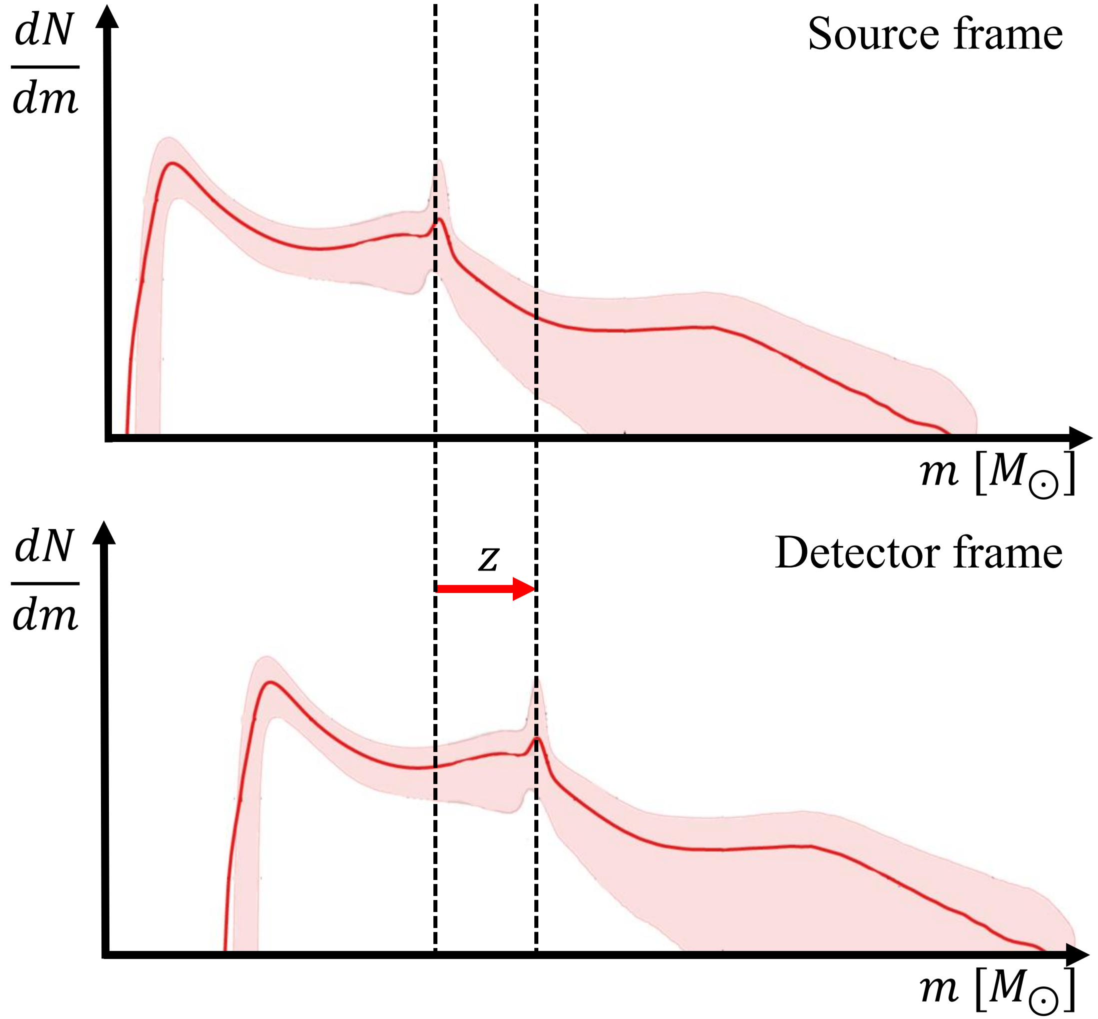
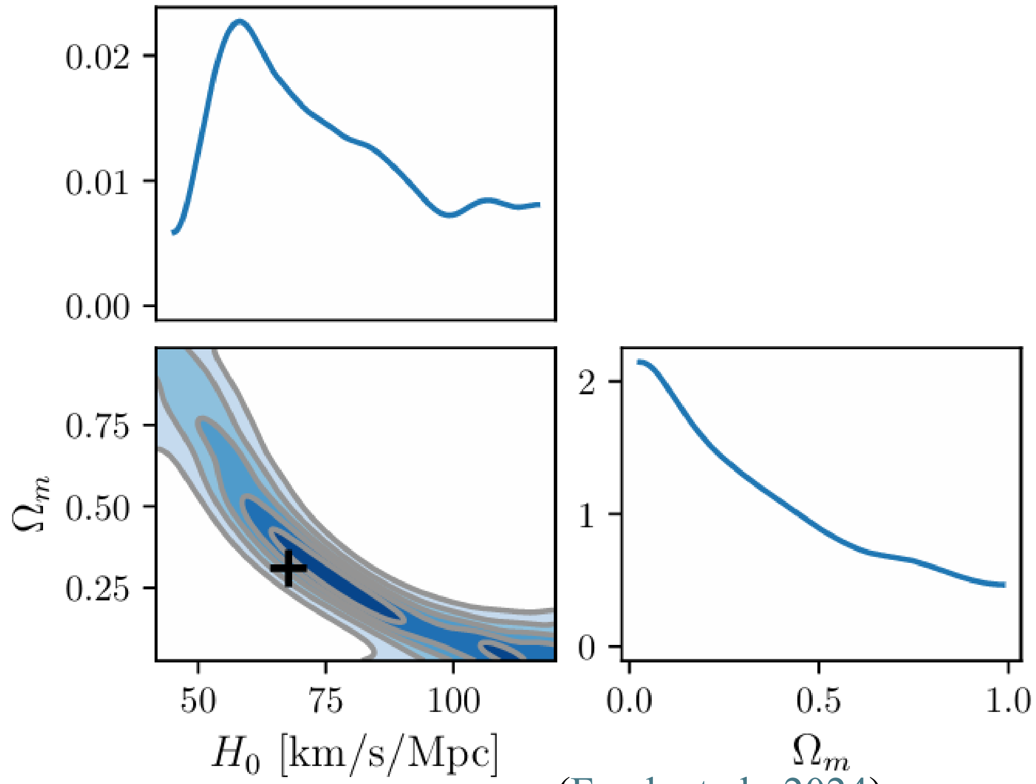


$$f_{max} \propto \frac{1}{M}$$

Higher detected mass because of Universe's expansion: $m_{det} = m_{source}(1 + z)$

Hubble constant with GW: redshift





(Farah et al., 2024)

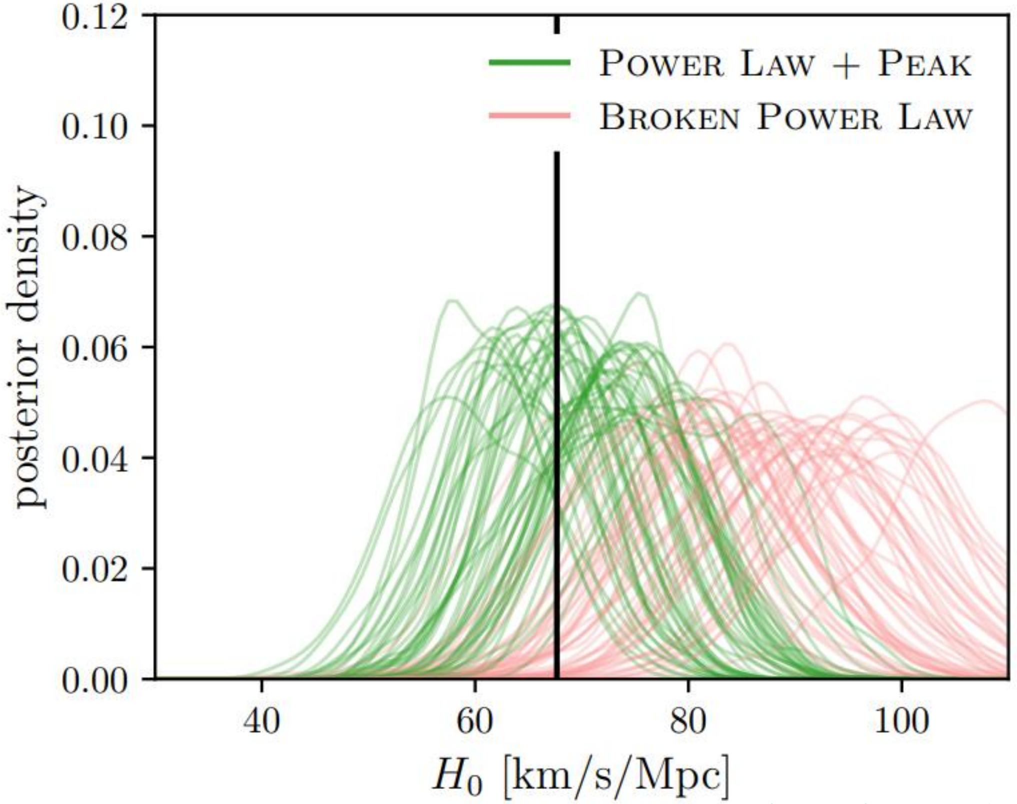
Hubble constant with GW: redshift

Strengths

- Much more common: 94 events up to O3
- Detectable at much higher redshift: constraints on $H(z)$
- Inference on astrophysical black hole population

Weaknesses

- Highly dependent on the assumed source-frame mass distribution



(Farah et al., 2024)

Conclusion/prospects

- Standard sirens can provide a third and independent way of measuring H_0
- Bright sirens will (hopefully) provide precise measurements of H_0 during O5
- Combining bright sirens to spectral and/or sirens will allow us to constrain $H(z)$ up to $z{\sim}3$ and study the astrophysics of compact binaries

Thank you

Bayesian statistics

$$p(A,B) = p(A|B)p(B) = p(B,A) = p(B|A)p(A)$$

Bayes theorem: $p(A|B) = \frac{p(A)p(B|A)}{p(B)}$

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$p(A|B)$: **posterior**

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$p(A|B)$: **posterior**

$p(B|A)$: **likelihood**

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$p(A|B)$: **posterior** $p(B|A)$: **likelihood** $p(A)$: **prior**

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$p(A|B)$: **posterior**

$p(B|A)$: **likelihood**

$p(A)$: **prior**

$p(B)$: **evidence**

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$p(A|B)$: **posterior** $p(B|A)$: **likelihood** $p(A)$: **prior** $p(B)$: **evidence**

$$p(H_0|D) = \frac{\pi(H_0)L(D|H_0)}{p(D)}$$

$$L(D|H_0) = L(D_{EM}|H_0)L(D_{GW}|H_0)$$

$D_{EM} \leftrightarrow z$ $D_{GW} \leftrightarrow h_+, h_\times$

Bayesian statistics: hierarchical model

$$h_+(f) \propto \frac{M_z^{5/6}}{D_L(z, H_0)} (1 + \cos^2(\iota)) f^{-\frac{7}{6}} e^{i\phi(M_z, f)}$$

$$h_\times(f) \propto \frac{M_z^{5/6}}{D_L(z, H_0)} \cos(\iota) f^{-\frac{7}{6}} e^{i\phi(M_z, f) + \frac{i\pi}{2}}$$

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Marginalization: $L(\mathrm{D_{GW}}|\mathrm{H_0}) = \int dD_L\, L(D_{GW}, D_L\,|H_0)$

$$p(H_0|D) = \frac{p(H_0)}{p(D)}\int dD_L dz\, L(D_{GW}|D_L(z,H_0))L(D_{EM}|z)p(D_L|z,H_0)p(z|H_0)$$

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$$H_0 = \frac{cz}{D_L}$$

$$H_0 = \frac{c(1+z)}{D_L} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_k (1+z')^2 + \Omega_\Lambda}}$$